Kuwait University | Math 101 | Date: November 9, 2006

Dept. of Math. & Comp. Sci. | First Exam | Duration: 90 minutes

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions:

1. Evaluate the following limits, if they exist:

(a) $$\lim_{x\to 1}\frac{[\tan 2(x-1)]^2}{x^2-2x+1}$$ (2 pts.)

(b) $$\lim_{x\to\infty}\left(\sqrt{2x^2+3}-\sqrt{2x^2-5}\right)$$ (2 pts.)

2. Find the vertical and horizontal asymptotes, if any, for the graph of

$$f(x)=\frac{|x-1|(1-2x)}{x^2+x-2}$$ (4 pts.)

3. Find the x-coordinates of the points at which the function f is discontinuous, where

$$f(x)=\frac{x^3+8}{x^2-x-6}$$

Classify the types of discontinuity of f as removable, jump, or infinite. (4 pts.)

4. (a) State The Intermediate Value Theorem. (1 pt.)

   (b) Show that the equation $x-\cos(\pi x)=0$ has a real solution. (3 pts.)

5. Use the definition of the derivative to find $f'(x)$, where $f(x)=x+\sqrt{x}$. (3 pts.)

6. (a) Let $y=\sqrt{3u^2+5u+3}$ and $u=2-\sec^2 x-\cot x$. Find $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$. (3 pts.)

   (b) Find $\frac{dy}{dx}$, where $y=[\cos(x^2-1)]^5$. (3 pts.)

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1. (a) $\lim_{x\to 1}\frac{[\tan 2(x-1)]^2}{(x-1)^2} = 4\lim_{x\to 1}\left(\frac{\tan 2(x-1)}{2(x-1)}\right)^2 = 4\left(\lim_{x\to 1}\frac{\tan 2(x-1)}{2(x-1)}\right)^2 = \boxed{4}$

   (b) $\lim_{x\to\infty}\left(\sqrt{2x^2+3}-\sqrt{2x^2-5}\right)\times\frac{\sqrt{2x^2+3}+\sqrt{2x^2-5}}{\sqrt{2x^2+3}+\sqrt{2x^2-5}} = \lim_{x\to\infty}\frac{(2x^2+3)-(2x^2-5)}{\sqrt{2x^2+3}+\sqrt{2x^2-5}} = \boxed{0}$

2. $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}\frac{|x-1|(1-2x)}{(x-1)(x+2)} = \boxed{\mp 2} \Rightarrow \boxed{y=-2 \,\&\, y=2}$ are $H.A$ for the graph of f.

   $\lim_{x\to -2^{\pm}} f(x) = \lim_{x\to -2^{\pm}}\frac{|x-1|(1-2x)}{(x-1)(x+2)} = \boxed{\mp\infty} \Rightarrow \boxed{x=-2}$ is $V.A$ for the graph of f.

3. $\lim_{x\to 3^{\pm}} f(x) = \boxed{\pm\infty} \implies f$ has an <u>*infinite discontinuity*</u> at $x=3$.

   $\lim_{x\to -2} f(x) = \boxed{-\frac{12}{5}} \implies f$ has a <u>*removable discontinuity*</u> at $x=-2$.

4. (b) $f(x) = x - \cos(\pi x)$ is continuous on $[0,1]$, $f(0) = -1 < 0$ and $f(1) = 2 > 0$. From The Intermediate Value Theorem, there is at least one $c \in (0,1)$ such that $f(c) = 0$.Thus, c is a solution of the equation $f(x) = 0$.

5. $f'(x) = \lim_{h\to 0}\frac{\left[(x+h)+\sqrt{x+h}\right]-\left[x+\sqrt{x}\right]}{h} = \lim_{h\to 0}\frac{h+\sqrt{x+h}-\sqrt{x}}{h}$

   $= \lim_{h\to 0}\frac{h}{h} + \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = 1 + \lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \boxed{1+\frac{1}{2\sqrt{x}}}$

6. (a) $u\left(\frac{\pi}{4}\right) = -1, \frac{dy}{du} = \frac{6u+5}{2\sqrt{3u^2+5u+3}} \,\&\, \boxed{\left.\frac{dy}{du}\right|_{u=-1} = -\frac{1}{2}}$

   $\frac{du}{dx} = -2\sec^2 x\tan x - \csc^2 x \,\&\, \boxed{\left.\frac{du}{dx}\right|_{x=\frac{\pi}{4}} = -6}$

   $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \left.\frac{dy}{du}\right|_{u=-1}\times\left.\frac{du}{dx}\right|_{x=\frac{\pi}{4}} = \left(-\frac{1}{2}\right)\times(-6) = \boxed{3}.$

   (b) $y' = 5\left[\cos(x^2-1)\right]^4\left[-2x\sin(x^2-1)\right].$